Turbulent scales smaller than the flame thickness

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We study in this paper the effect of turbulent scales smaller than the premixed flame thickness on flame propagation and structure. We compare our numerical results to existing theories of flame broadening, showing that the apparent Lewis number tends to unity for very high forcings, but that the small scale forcing can generate a large-scale curvature of the front. [S1063-651X(99)02003-6]

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I. INTRODUCTION

The problem of premixed turbulent flames is one of the most important in combustion, and has a wide range of applications in various combustion devices, including internal (spark ignition) combustion engines. An aspect of the problem is the wrinkling of a flame by vortices larger than the flame thickness, which leads to the notion of turbulent flame speed, typically much larger than the laminar velocity. This has been the subject of a large amount of work, and in this case the flame can be described in a simple way as a discontinuity propagating normally with a given laminar velocity, apart from curvature corrections, and submitted to a turbulent flow field [1]. It is customary, in the case of high turbulence amplitudes, to neglect the retroaction of the flame on turbulence, and indeed it has been shown in Ref. [2] that the fractal dimension of the front is not very much modified by including gas expansion effects, which lead to the well-known Darrieus-Landau instability of premixed flames.

However, in many industrial applications, the turbulent forcing is so high that velocities at scales below the flame thickness can no longer be neglected. In this case, although it seems that large heat losses are necessary [3], it often happens that flames can be extinguished [4–6] and this is naturally a very important phenomenon in applications, which limits the operating conditions of many combustion facilities. The role of small scales in the extinction phenomenon is difficult to assess, because, although they lead to very high strain rates, their correlation time is small and limits in a very important way their effectiveness in extinguishing the flame. There has been some debate about this point in the literature [7,8].

Nevertheless, even if the scales of turbulence below the flame thickness do not succeed in extinguishing the flame, the phenomenon of flame broadening is important in itself and will be the subject of this paper. A theoretical study of this broadening has been performed in Ref. [9], and it has been suggested in this paper that the main role of turbulence inside the flame thickness is to renormalize the laminar flame velocity. The problem of turbulence above the flame thickness can now be handled by considering that the flame is a discontinuity, but propagating with this new laminar velocity. The authors thus consider that a separation of scales between scales above and below the flame thickness is possible in general. They make use of the idea of locality in wave-vector space [10]: a vortex at a certain scale is supposed to curve the front, or modify the flame thickness, at the same scale. This idea of locality in scale space has been questioned in Ref. [11], where it has been shown, on the basis of an equation describing the propagation of a discontinuity, that small scales of turbulence can wrinkle the front at large scale. In this paper, on the contrary, we will study flames with a finite thickness, and see if it is really easy to separate the effects of small and large scales on the flame behavior.

Another point of interest of Ref. [9] concerns the apparent Lewis number seen in turbulent flames with scales below the flame thickness. The Lewis number is usually defined in combustion as the ratio of the thermal to the molecular diffusivities, and we will call in the sequel apparent Lewis number the ratio of a flame thickness defined by the temperature field to a flame thickness defined by a concentration field (the concentration of the limiting reactant). According to Ref. [9], the apparent Lewis number should tend to 1 for sufficiently high-turbulence forcings. The idea is very simple: the turbulence inside the flame thickness generates a turbulent diffusivity which should be the same for temperature and concentration. For sufficiently high forcings, this turbulent diffusivity should dominate the laminar diffusivities and thus the apparent Lewis number should be close to 1.

Nevertheless, a paper has appeared suggesting that, on the contrary, the apparent Lewis number should depart further from 1 for high-turbulence amplitudes (increase for $Le>1$, decrease for $Le<1$) [12]. This surprising theoretical result is obtained with a model which is supposed to be a direct extension of Ref. [9]. We will test this idea in our simulations.

The plan of the paper will be as follows. In Sec. II, we present the model used in the numerical simulations. In Sec. III, we investigate the problem of flame broadening and apparent Lewis number, in order to see if this Lewis number tends towards 1 or not for high forcings. In Sec. IV, we show some qualitative results suggesting that the role of scales below the flame thickness on the global flame behavior cannot be reduced in general to a renormalization of the laminar flame velocity. Finally, Sec. V contains a short conclusion.

II. MODEL

In order to simplify the problem of flame turbulence interaction, we will work in the framework of the thermal-diffusive approximation. This approximation considers that gas expansion is small, so that the thermal diffusive variables—temperature T and mass fraction of the limiting reactant Y—evolve in an imposed flow field $\mathbf{v}$. Thus with this approximation there is no hydrodynamic instability, as there is no retroaction of the thermal-diffusive variables on the flow field.

Actually this approximation is equivalent to the one usually made for scales larger than the flame thickness when one solves the problem of a discontinuity propagating normally with a given velocity, but submitted also to a turbulent flow field without retroaction. This is usually done by using an eikonal equation describing flame propagation [1], i.e., by transforming the front propagation problem into a field equation in order to avoid the difficult problem of reconnections occurring on the front. As shown in Ref. [13], it is possible to solve the same problem in a purely lagrangian way, and even to include the hydrodynamic instability [2] by solving an integral equation written by Frankel [14].

As we are interested in this paper in the role of scales below the flame thickness, we are obliged to solve the two field equations for T and Y, which in the thermal-diffusive approximation can be written as

$$\frac{\partial T}{\partial t}+\mathbf{v}\cdot\boldsymbol{\nabla}T=\Delta T+\Omega,$$

$$\frac{\partial Y}{\partial t}+\mathbf{v}\cdot\boldsymbol{\nabla}Y=\frac{1}{\mathrm{Le}}\Delta Y-\Omega,$$

where T and Y are normalized so as to vary from 0 to 1 (0 in in fresh gases and 1 in the burnt gases for T, the opposite for Y), Le is the Lewis number (ratio of the thermal to the molecular diffusivities), and Ω is the production term:

$$\Omega=\frac{\beta^2}{2\mathrm{Le}}Y\exp\left(\frac{\beta(T-1)}{1+\gamma(T-1)}\right),$$

where $\gamma=0.8$ and β (the Zeldovich number) is a reduced activation energy. We will take in the simulations the relatively low value $\beta=5$, which leads a reaction term that is not very stiff.

It remains to specify the turbulent velocity field $\mathbf{v}$. For a square domain of length $L=5$ (in units of flame thickness), we will take

$$v_x=-\sum_{k_i} ak_i^{-5/6}\cos(k_ix+\varphi_{ix})\sin(k_iy+\varphi_{iy})\cos(\varphi_{it}),$$

$$v_y=\sum_{k_i} ak_i^{-5/6}\sin(k_ix+\varphi_{ix})\cos(k_iy+\varphi_{iy})\cos(\varphi_{it}),$$

where a controls the amplitude of the turbulent field. The possible wave vectors in the computational domain are $k_i=(2\pi i)/L$, where i is an integer number ranging from 15 to 25. φ_{ix}, φ_{iy}, and φ_{it} are constant random phases associated with the wave vector k_i. This velocity field is just an example of an incompressible velocity field involving different scales all below the flame thickness (between $\frac{1}{3}$ and $\frac{1}{5}$ of the laminar flame thickness). The results obtained in this paper are apparently independent of the precise form of the flow field chosen. We will work in the sequel with constant cutoffs and vary the amplitude of the forcing.

Let us note that the flame is constantly advancing, and that as a consequence, the flow field seen by the front changes with time. We have to modify at each time step the velocities of the mesh points in order to be able to keep the mean position of the front at a constant place in the computational mesh. The results of Sec. IV will be presented in this reference frame moving with the front. Let us also say that we use a typical resolution of 240 points in the longitudinal direction with a finite difference discretization, and 128 Fourier modes in the transverse direction, with a Fourier pseudospectral discretization. The longitudinal direction is naturally defined to be the direction parallel to the direction of the mean propagation. The discretization in time is first order accurate.

III. FLAME BROADENING AND APPARENT LEWIS NUMBER

In this section, we address the problem of the apparent Lewis number of a flame submitted to turbulence below the flame thickness. As recalled in the Introduction, two contradictory theories have appeared concerning this point [9,12]. In the first paper is presented the usual view that the apparent Lewis number tends towards 1 for high forcings, as the turbulent diffusivity (the same for temperature and concentration) dominates in this case. In Ref. [12], on the contrary, the authors present some reasoning showing exactly the opposite: according to them, the apparent Lewis number departs further from 1 when the forcing is increased.

We compare in this section the results of our numerical simulations to these predictions. We perform simulations for different values of the amplitude, and show curves showing the different thicknesses (for temperature and concentration) and the apparent Lewis number.

But first, how do we define these quantities? We recall that temperature and concentration are normalized so as to vary from zero to one. Naturally, as is well known, there is no unique way to define a flame thickness. We use here the following definitions: the flame thickness for temperature is defined as the difference in position between the temperature line 0.331 and the temperature line 0.9; the thickness for concentration (mass fraction actually) is defined as the difference in position between the value $Y=0.669$ and the value $Y=0.1$. We define a concentration thickness and a temperature thickness for each value of the transverse coordinate, and we define the mean of these quantities over all possible values of the transverse coordinate. Thus we obtain what we will call concentration and temperature thicknesses in the text. The apparent Lewis number is now simply defined as the ratio of the temperature thickness to the concentration thickness.

Naturally, as we use a relatively low value of the Zeldovich number in the simulations, the reaction zone is relatively large, and the values of the apparent Lewis number are different from the true Lewis number even for zero forcing. We would obtain an apparent Lewis number for zero forcing close to the true value only for a very high value of the activation energy. Of course it is not possible to use large values of activation energy and Lewis numbers different

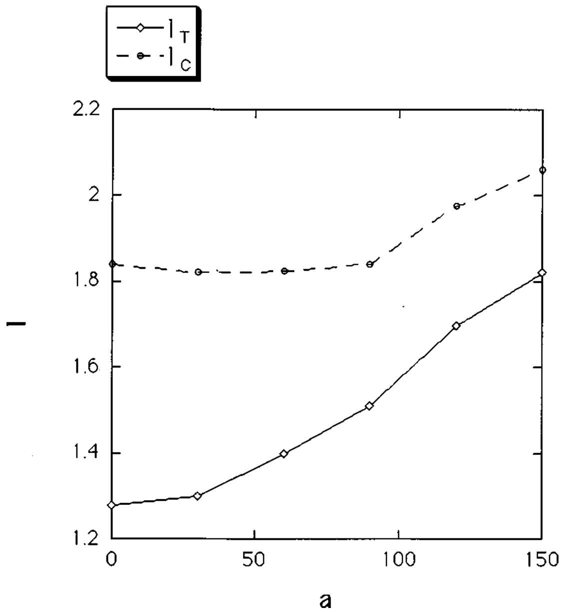

FIG. 1. Concentration (l_C) and temperature (l_T) thicknesses versus a for $\beta=5$ and Le$=0.6$.

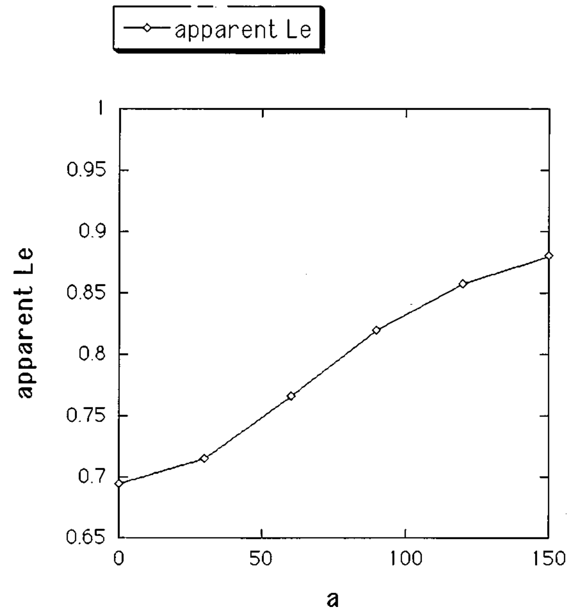

FIG. 2. Apparent Lewis number versus a for $\beta=5$ and Le$=0.6$.

from 1 because it would lead to the thermal-diffusive instability of the plane front. With the parameters chosen, we emphasize that the plane front will always be stable: the solution stays plane at zero forcing. Now we are in a position to see how the two thicknesses evolve and the apparent Lewis number with increasing amplitudes.

In Fig. 1, we show the thicknesses relative to temperature and concentration versus the amplitude (the parameter a appearing in the definition of the velocity field) for a Lewis number Le$=0.6$. As the Lewis number is lower than 1, the temperature thickness is smaller than the concentration thickness. But, although both thicknesses increase with the forcing (the phenomenon of flame broadening), it appears that the temperature thickness increases much faster than the concentration thickness. Thus the difference between the two thicknesses is reduced for high forcings. The amplitudes necessary to see this effect are very high. They correspond to a velocity for a given wave vector of the order of several laminar flame speeds. However, because of the small correlation times, the flame speed is not very much increased by the forcing, and, for instance, the flame speed is only 1.5 for $a=150$ and Le$=0.6$ (where 1 is the laminar flame velocity).

As the flame is advancing through the turbulent field, there are large fluctuations occurring on the front, and the thicknesses shown in Fig. 1 fluctuate in time in a relatively important way, although they already represent a spatial mean, as explained at the beginning of this section. So the errors involved in the measurement are not negligible in general. A rough estimation of this kind of error can be ±0.03, but naturally it increases with the forcing. However, when one calculates the ratio of both thicknesses in order to obtain the apparent Lewis number, the fluctuation in time is reduced (typical error ±0.01), and the curve shown in Fig. 2 is obtained also for the case Le$=0.6$. As can be seen in this figure, this apparent Lewis number increases towards a value close to 1. So we obtain a result in agreement with the usual view that the apparent Lewis number must be close to 1 for high enough forcings [9], and not with the predictions of Ref. [12]. The forcings necessary to see this effect, however, are very high.

In Fig. 3, we study the same kind of effect in the case of a Lewis number larger than 1, i.e., Le$=1.6$, and we show the two thicknesses versus turbulence amplitude. Now the concentration thickness is smaller, but increases with forcing in a faster way than the temperature thickness. If one compares Fig. 3 with Fig. 1 (the same figure but with Le$=0.6$), it is seen that both thicknesses increase faster in the Le$=1.6$ case. In Fig. 4 is shown the apparent Lewis number in the Le $=1.6$ case, and now this apparent Lewis number decreases towards 1. Thus in both cases Le$=0.6$ or 1.6 we have the same effect: the turbulent diffusivity dominates for high forcings.

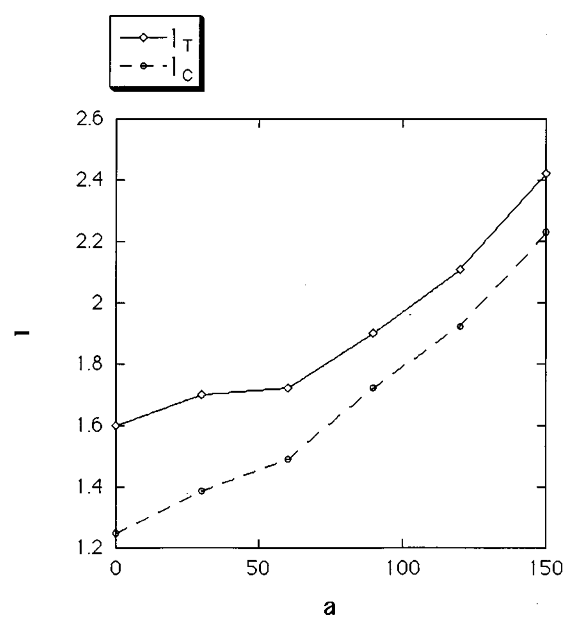

FIG. 3. Concentration (l_C) and temperature (l_T) thicknesses versus a for $\beta=5$ and Le$=1.6$.

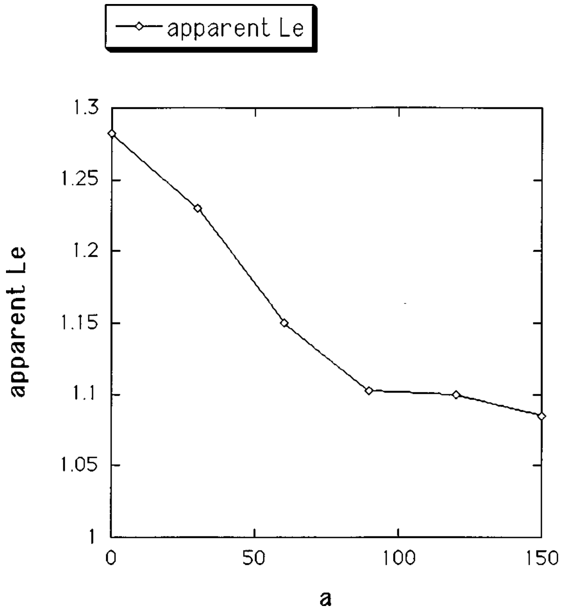

FIG. 4. Apparent Lewis number versus a for $\beta=5$ and Le$=1.6$.

IV. LARGE-SCALE CURVATURE CAUSED BY SCALES BELOW THE FLAME THICKNESS

In this section, we show qualitative results on typical aspects of the temperature, concentration, and production fields during their evolution in time. These results are relatively surprising, but let us begin by recalling how the effects of the scales below the flame thickness on the scales larger than this thickness are usually taken into account [9]. Viewed from the point of view of large scales, the main property of the premixed flame is that it is advancing in time with a given laminar velocity. The real cause of this advance is of course the coupling of reaction and diffusion inside the flame thickness. Now, if turbulence below the flame thickness is included, a turbulent diffusivity must be accounted for in the analysis of what occurs inside the thickness. But it is usually believed that the only effect on the large scales is to change the value of the laminar velocity.

Before showing the figures of the different fields, we would like to insist on a very important point: all the simulations are performed here with parameters chosen in order for the plane front to be stable relative to the thermal diffusive instability.

We show in Fig. 5 a solution corresponding to $\beta=5$, Le $=0.6$, $a=120$. In Fig. 5(a) is shown the temperature field, in Fig. 5(b) the concentration field, and in Fig. 5(c) the production term. Different equidistant temperature lines, for instance are shown in Fig. 5(a). The fresh gases are on the left and the flame is advancing in this direction.

It is seen that the different fields calculated are extremely corrugated at small scale because of turbulence. As the Lewis number is smaller than 1, the concentration thickness [Fig. 5(b)] is larger than the temperature thickness [Fig. 5(a)]. But an important property seen in these figures is that the flame is curved at large scale (here at the largest wavelength available), although there is no forcing larger than the flame thickness. This surprising result is particularly obvious in Fig. 5(c), where the production term is higher in the zone

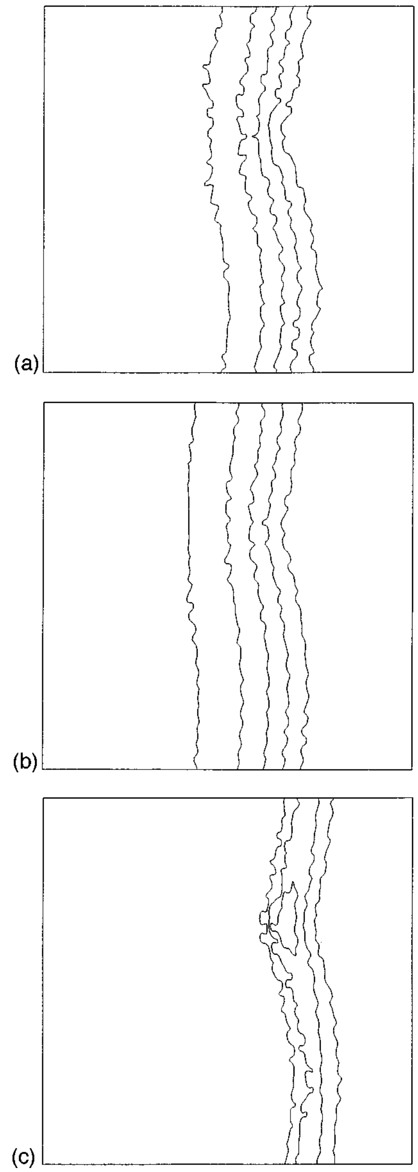

FIG. 5. Solution for $\beta=5$, Le$=0.6$, $a=120$. (a) Temperature, (b) mass fraction, and (c) production term.

pointing towards fresh gases, resulting in a higher flame velocity in this zone.

As the discretization used in the transverse direction is pseudospectral, there is an aliasing error present in the results of the simulation. This type of error could participate in the large-scale wrinkling observed. However, the cutoff at small scale of the turbulent field is much larger than the smallest scale on the mesh and we have verified that increasing the number of modes in the transverse direction (and thus reducing the aliasing error) does not change the large-scale wrinkling. This wrinkling is also observed for different values of the Lewis number, including Le$=1$.

So we have here a situation where the role of small scales cannot be reduced to only a renormalization of the laminar velocity. Actually a similar effect, showing that a turbulence at a certain scale does not necessarily curve the front at the same scale, has been observed in Ref. [11], in the framework of a model equation describing the propagation of a discontinuity. In this paper, all scales were larger than the flame

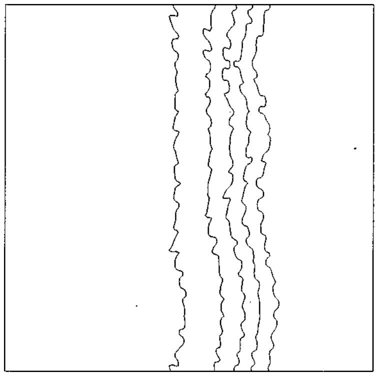

FIG. 6. Temperature field for $\beta=5$, Le$=0.6$, $a=120$ at a time step later than Fig. 5.

thickness. What is surprising here is that we have a similar effect, although the scales of turbulence are all below the flame thickness. And as in Ref. [11], the large scales observed are the largest possible, although the solution oscillates in time and can exhibit at a particular time step a dominant wavelength that is half the width of the domain. Such a case is shown in Fig. 6, where the same solution $\beta=5$, Le $=0.6$, $a=120$ is shown some time later (only the temperature field is shown here). The large scales are created by the nonlinear interaction of very close wave vectors. Other situations where small-scale forcing creates large-scale motion have been described in the literature (see, for example, Ref. [15]), where this large-scale motion was shown to be created by a negative eddy viscosity. It is difficult to know here whether the large-scale wrinkling can be described by an effective negative Markstein length or by a renormalization of the forcing at large scale.

The importance of this wrinkling could depend on correlation times, as shown in Ref. [11], and although this effect seems to be very general, it is not impossible that it is relatively small in experimental situations. Tests of this idea are needed, but it must be realized that it is difficult to separate the effect of scales above and below the flame thickness in an experiment. Perhaps experiments involving aqueous autocatalytic reactions [16,17] offer the best hope of carefully controlling the role of the different scales. For instance, in this case, there is no hydrodynamic instability which could participate in the wrinkling at the largest scales.

V. CONCLUSION

In this paper, we have studied the role of scales of turbulence below the flame thickness on flame propagation. It appears that this kind of turbulence widens the flame and that the apparent Lewis number measured by taking the ratio of the temperature to the concentration thickness becomes close to 1 for high enough forcings. But in this range of forcings, a relatively surprising phenomenon is observed: these very small scales of turbulence are able, in the model solved, to create a wrinkling of the flame at a scale much larger than the flame thickness. This effect is not very compatible with the usual view that the only role of scales below the flame thickness is to define a new laminar burning velocity. It remains to be seen whether this effect can be observed in the case of realistic turbulence.

[1] A. R. Kerstein, W. T. Ashurst, and F. A. Williams, Phys. Rev. A **37**, 2728 (1988).
[2] B. Denet, Phys. Rev. E **55**, 6911 (1997).
[3] I. Brailovsky and G. Sivashinsky, Phys. Rev. E **51**, 1172 (1995).
[4] R. G. Abdel-Gayed, D. Bradley, and F. K.-K. Lung, Combust. Flame **76**, 213 (1989).
[5] J. Chomiak and J. Jarosinski, Combust. Flame **48**, 241 (1982).
[6] T. Poinsot, D. Veynante, and S. Candel, in *Twenty-third Symposium (International) on Combustion* (The Combustion Institute, Pittsburgh, 1990).
[7] C. Meneveau and T. Poinsot, Combust. Flame **86**, 311 (1991).
[8] A. Yoshida, H. Kakinuma, and Y. Kotani, in *Twenty-sixth Symposium (International) on Combustion* (The Combustion Institute, Pittsburgh, 1996).
[9] P. D. Ronney and V. Yakhot, Combust. Sci. Technol. **86**, 31 (1992).
[10] A. Pocheau and D. Queiros-Conde, Phys. Rev. Lett. **76**, 3352 (1996).
[11] B. Denet, Combust. Theory Modelling **2**, 167 (1998).
[12] M. Dulger and E. Sher, Combust. Sci. Technol. **119**, 1 (1996).
[13] B. Denet, Combust. Sci. Technol. **123**, 247 (1997).
[14] M. L. Frankel, Phys. Fluids A **2**, 1879 (1990).
[15] G. I. Sivashinsky and A. L. Frenkel, Phys. Fluids A **4**, 1608 (1992).
[16] S. S. Shy, R. H. Jang, and C. Y. Tang, Combust. Flame **105**, 54 (1996).
[17] B. D. Haslam and P. D. Ronney, Phys. Fluids **7**, 1931 (1995).